

This article is the translation of “Application d’outils fractals pour la classification d’images microscopiques de matières grasses laitières” that has been presented at the GRETSI in 2005.

# Application of fractal tools for the classification of microscopical images of milk fat

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**Abstract** – Numerous butter features, such as "spreadability", depend on the crystallization, which is observable on microscopical images of milk fat. As microstructures often reveal rough or irregular aspect, fractal tools appear to be particularly adapted to their study.

In this paper, fractal techniques quantifying the texture of the image are applied to separate type, plant and day of ripening of different butters. The images are taken from butters produced in industrial conditions. The influence of the crystallization on the fractal parameters has been observed. Fractal quantifiers have demonstrated promising correlations with the day of ripening of the butter. Moreover, they have also shown to be efficient in separating different kinds of butters.

## 1 Introduction

Numerous works of milk engineering focus on the development of technological ways of improving the functional and nutritive properties of butter milk fat. The aim of the studies is the production of butter with lower cholesterol content for the preparation of "health milks", of so-called frigo-spreadable butters, of low-fat butters and of butters with improved functionality for specific uses (e. g. bakery or chocolaterie trade).

All these properties mainly depend of the microstructure, which is traditionally measured through rheology but can also be observed on some microscopical images. These images often present an irregular aspect. The use of fractal tools seems therefore suitable to the analysis of this kind of images.

Works on fractal analysis of gels and fat matters (coco butter, oils, yogourts, cheeses) are generally based on models linking microscopic and macroscopic properties of the matter: Shih [6] develops a model that links the elastic properties to a fractal dimension of the flocks in the colloidal gel. Wu and Morbidelli [7] propose a scaling theory that links the structural parameters with the rheological properties. Marangoni [4] evaluates a fractal dimension of the microscopical images of the fat matter in order to modelize elastic properties. In these articles, the fractal dimension is a parameter in the analytic model that helps modeling rheological properties.

In this work, we use fractal techniques independently of a modeling based on the rheology and evaluate the crystallisation of the fat matters on the images. An advantage of this method as compared to classical penetrometry methods is that it is easier to implement. Moreover, it is applicable from the day the butter is made. The classical penetrometry technique cannot be used at this date because the butter at this date is too soft. We use several fractal parameters, among which the most pertinent have been found to be the box dimension, the regularization dimension, the lacunarity and some Besov norms.

This article is structured as follows : Section 2 describes the employed material and acquisition methods, section 3 presents

in a general way the fractal parameters that have been retained for the study, section 4 exposes and discusses the obtained results, and section 5 concludes the article.

## 2 Material et methods

We worked with microscopical images of two kind of butters: french croissant butter (used, as its name indicates, in the preparation of croissants), and Nizo butter. Nizo butter is obtained by a method of continuous maturation of the cream. It covers 90% of the industrial production of butter. Samples of the two types of butter have been taken from several industrial sites (two for Nizo butter, three for Croissant butter). The samples are taken on the sites when they come out of the conditioners and carried in isotherm containers to the ENILIA<sup>1</sup> facilities. The samples are then followed during their 21-days maturation. For each type of butter and each site, images are taken at days J0, J+1, J+7, J+14, J+21, where J0 is the day of production of the butter. Each day, five images of the sample are taken, for two or three slide thicknesses. The magnification of the microscope is 400x (1 pixel  $\leftrightarrow$  1.18  $\mu$ m). Thus, we had 60 images of size 1104x812 pixels by butter and by site to our proposal.

On the images (Fig. 1), one can see crystals assuming the shape of a Maltese cross (lighter shades of gray) and a medium gray, shapeless mass, corresponding to non-crystallized matter, or to superposed crystals. The crystals are particularly visible on the images of croissant butter. Depending on the type of butter, the crystals appear more or less clearly. As the maturation goes on, the crystallization increases, which causes the shapeless mass to vanish and the crystals to become bigger and less numerous.

To summarize, the features we shall concentrate on to differentiate the butters are: The butter type, its maturation degree

<sup>1</sup>Organism which performed this work with us: Ecole Nationale d'Industrie Laitières et des Industries Agro-alimentaires. Site : <http://www.enilia.com>

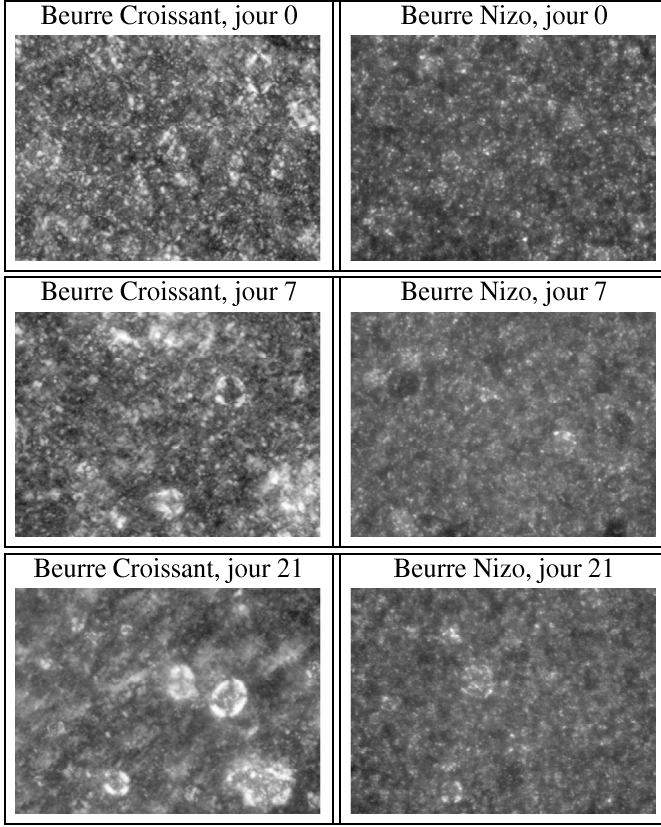


Figure 1: Images de beurre

and its production site. In that view, we shall use fractal tools.<sup>2</sup>

## 3 Presentation of fractal tools

### 3.1 Box dimension

The box dimension [2] is a classical fractal parameter that allows to characterize the way that an object fills the space. It has been computed on black-and-white pictures (The black and white pictures are obtained by threshold on the grayscale images. See paragraph 4.1 for more details). One first computes a sequence  $N(k)$  defined as follows: For each  $k$ , one covers the image with a grid of squares with length  $1/2^k$ .  $N(k)$  is the number of non-empty squares (i.e. containing at least one white pixel).

The box dimension measures the speed at which  $N(k)$  tends to  $\infty$ . Roughly speaking,  $N(k)$  is proportional to  $2^{kd}$  where  $d$  is the searched for dimension. More precisely,  $d = \lim_{k \rightarrow \infty} \frac{\log N(k)}{\log 2^k}$  if this limit exists.

The goal is to evaluate  $d$ . In that view,  $\log N(k)$  is plotted as a function of  $\log k$ . Then an interval on which this plot seems to be affine is determined.  $d$  is the estimation of the slope of the obtained regression line.

Note that the same type of calculation may also characterize grayscale image (seen as a surface in  $\mathbb{R}^3$ ), but that the results have not been interesting in our application.

### 3.2 Regularisation dimension

Another fractionnal dimension, called “regularization dimension” [5], has given interesting results. It one was computed on the grayscale image: The goal is not anymore to characterize the way that some pixels fill the image, but to get an information on the regularity of its texture.

Let  $I(x, y)$  the image, where  $I(x, y)$  is the gray level of pixel  $(x, y)$ .  $I$  is smoothed by convolution with, for example, a gaussian kernel  $K_\sigma$  of variance  $\sigma$ . Let  $I_\sigma$  denote the regularized image  $I_\sigma = K_\sigma * I$ . Suppose that  $I$  is so irregular that the surface  $(x, y, I(x, y))$  of  $\mathbb{R}^3$  has an infinite area.

The images  $I_\sigma$  are such that for every  $\sigma > 0$ , their surfaces  $(x, y, I_\sigma(x, y))$  have a finite area  $S_\sigma$ . When  $\sigma$  tends to 0,  $I_\sigma$  tends to  $I$  and  $S_\sigma$  tends to infinity. the regularization dimension is defined as :  $\dim_R = 2 + \lim_{\sigma \rightarrow 0} \frac{\log(S_\sigma)}{-\log \sigma}$  if the limit exists (else, an lower limit is evaluated).

As for the box dimension, the regularization dimension is evaluated through a linear regression of  $\log S_\sigma$  as a function of  $\log \sigma$ .

### 3.3 Lacunarity

Two images with the same fractionnal dimension may differ tremendously. The lacunarity [3] is a second-order fractal parameter that describes the texture of an image. In a same way as the regularization dimension, we applied it on the grayscale images.

The lacunarity evaluates the homogeneity of the repartition of the luminosity. If the image is supposed to be totally homogeneous, the luminosity in a window of size  $\epsilon$  is the same for every window. The lacunarity describes the “dispersion” of the luminosities really present in these windows of size  $\epsilon$  as compared to the mean luminosity  $m$  in a window of size  $\epsilon$ :  $L = \langle (\frac{m'}{m} - 1)^2 \rangle$  where  $m'$  is the luminosity observed in the window of size  $\epsilon$ . The brackets design a mean over all the windows of size  $\epsilon$ .

The lacunarity allows to measure the distribution of the holes in the image: If it contains big contrasts (at scale  $\epsilon$ ), then the image lacunarity will be high. On the contrary, if the image is homogeneous, then its lacunarity will be low.

### 3.4 Besov norm

We explain the computation in 1D for clarity. Let  $X$  a signal in  $L^2(\mathbb{R})$  and  $c_{jk}$  be its wavelet coefficients on a basis with a sufficient regularity (see [5] for more details). Then  $X$  belongs to the Besov space  $B_{p,q}^s$  if :

$$\|X\|_{B_{p,q}^s}^q = \sum_j \left[ 2^{j(s+1/2-1/p)} \left( \sum_k |c_{jk}|^p \right)^{1/p} \right]^q < \infty$$

The parameter  $s$  measures the signal regularity, and parameters  $p$  and  $q$  the way of defining this regularity.

## 4 Discussion and analysis

Crystals whose aspect varies with the butter type and the maturation stage can be observed figure 1. The parameters that have been exposed in section 3 may be interpreted as reflecting some properties of the crystals such as their size, their density or the quantity of non-crystallised matter.

<sup>2</sup>These tools are available in the toolbox FRACLAB, downloadable at: <http://www.irccyn.ec-nantes.fr/irccyn/Projets/FracLab/FracLab.htm>

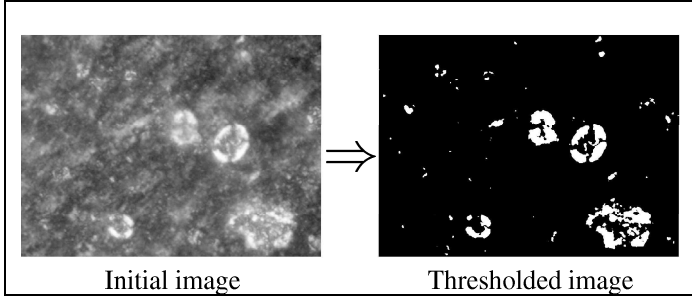


Figure 2: Example of thresholding

We have tried to discriminate the different butter groups using these fractal parameters. The interest of this method is the following : a tool that distinguish the days gives information on the maturation state of the butter, even in the absence of model. A tool that distinguishes the butter types may allow to verify that a butter is in keeping with what one wants to get. At last, discriminating the production sites may allow to highlight variations of technology of raw materials.

Each one of the previously exposed parameters has been studied in the frame of the separation of butter type, of production place and of day of follow-up. We present here the more concluding results: each parameter was the most efficient in one kind of separation.

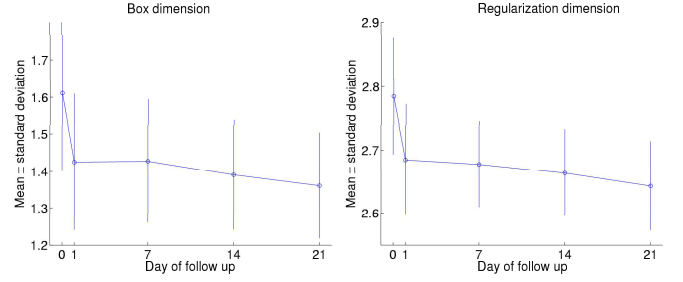
#### 4.1 Evolution of the box dimension with the maturation

Before computing this parameter, we threshold. The goal is to get images where white pixels correspond to crystals, these crystals being clearer than the background. Because of the number of processed images, this threshold had to be fixed in a automatic way. The computation of the threshold is a problem since the mean luminosity and the contrast of the image depend of the lighting during the photography. As this lighting may vary, it seems necessary that the binarised image depends as less as possible on these two parameters.

We have observed that an empirical threshold fixed to the mean + 2 standard deviations allowed to isolate the surface occupied by the crystals in a satisfactory way. We verified on a high number of images that this type of threshold was indeed correct.

On the images of the first days, the crystals are small and uniformly distributed. Seen from far away, the image may appear as full of crystals. When crystals aggregate, their surface increases, but there is more space between them and, at the condition of still looking to the image from far enough, they appear more as some "points".

The box dimension allows to distinguish these changes: Assume for instance that small crystals are everywhere in the image. Then every box will be non-empty, so that  $N(k)$  values an  $2^{2k}$ . On the other hand, if the image contains just isolated crystals whose size remains smaller than the box sizes, then there will be always a non-empty box by crystal and  $N(k)$  will be constant, equal to the crystal number. The real situation is in between these extreme cases, but one has been able to observe that the box dimension decreases with the day of maturation (Fig 3(a)). Precise that the analysed images are images of butter prepared in industrial conditions. The results are rep-



(a) Box dimension of croissant butter (b) Regularization dimension of croissant butter

Figure 3: Evolution with the day of follow-up

resentative, each mean being evaluated on 180 images (for the three sites) despite relatively high error bars.

#### 4.2 Evolution of the regularization dimension with the butter maturation

During butter maturation, the formation of new crystals, the agregation of already existant crystals and the vanishing of shapeless mass can be observed. As the crystals are a relatively regular part of the image, contrary to the shapeless mass, the irregularity of the image decreases with time. The regularization dimension should to diminish with the butter crystallisation. The results on croissant butter Fig. 3(b) show that the regularization dimension is indeed correlated with the butter maturation and decreases with time. As previously, the results are representative because of the large number of analysed images (180 a day).

#### 4.3 Separation of butter types with lacunarity

The lacunarity evaluates the homogeneity of the image texture. It may therefore allow to separate the two types of butter: The croissant butter images entails numerous empty spaces and are not homogeneous, because of the presence of big crystals in the shape of a Maltese cross. In comparison, the Nizo butter images are much more homogeneous. The lacunarity values of the croissant butter are in deed much smaller than those of the Nizo butter. Figure 4(a) shows that lacunarity separates perfectly both type of butter on all the samples (approximately 360 by butter type). Moreover, it can be seen that the concentration of the samples is low for the critical value  $L = 0.06$ . The lacunarity seems thus to be well adapted to the butter type separation.

#### 4.4 Besov norm and production plant

The separation of the several industrial sites has been done using Besov norm of the images in well-chosen Besov spaces [1]

The choice of these spaces has been done in this way: we computed the Besov norm in 45 different spaces (for  $s = 0.3, 0.4, 0.5$ ;  $p = 1, 2, 3$ ;  $q = 1, 2, 3$ ). Having observed that the results were partitionned into two groups of results inside which the correlations were particularly strongs, we kept a representative of the group that discriminated the better the two sites.

We have been able to separate the sites that where producing the Nizo butter (See figure 4(b)), but not the croissant butter.

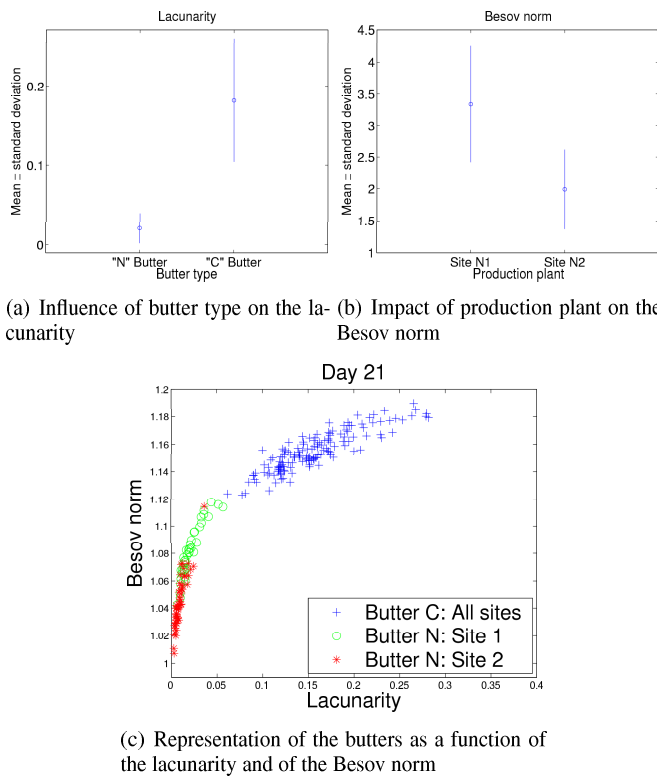


Figure 4: Separation of butter type and production plant

The results have thus revealed to be complementary to the penetrometry results : with statistical tools like principal component analysis on classical penetrometry measures, it has been possible to separate the production site of croissant butter, but not those of Nizo butter. Besov norms have also shown to be efficient combined with other fractal parameters. Figure 4(c) presents the classification by site and by butter type as a function of the lacunarity and of Besov norm; this combined criterium allowed to separate the croissant butter from the Nizo butter, and Nizo butters among the plant in which they were made. We have been studying several factor combinations. The couple (Besov, Lacunarity) is the most interesting one. The interest of this separation is for example to distinguish non-conform butters.

## 5 Conclusion

Fractal techniques seem to be well fitted to evaluate the maturation of Nizo and croissant butters, to separate the two types of butter and discriminate industrial plants where the Nizo butters are made. Unlike penetrometry methods, the results are immediately available; we can thus quantify the crystallization stage from the first day of maturation. Moreover, the image analysis tools are easier to implement

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